PATH COUNTING

CROSSROADS ACADEMY
MATHCOUNTS PREPARATION

I) If a king begins at the lower left square of a standard chessboard, how many paths can he take to the upper right square if he can only move up or right? Can you find a formula for a general $n \times n$ chessboard?

II) If a knight begins at the lower left square of a standard chessboard, how many paths can he take to the upper right square if he can only move up and right or right and up? Can you find a formula for a general $n \times n$ chessboard?

III) If a rook begins at the lower left square of a 2×4 chessboard, how many paths can he take to the upper right square if he can only move up or right? Can you find a formula for a general $2 \times n$ chessboard?

IV) Consider paths moving only up or right on a 7×3 grid, beginning at the lower left square. What is the smallest positive integer that does not occur as a possible number of paths to a point?

Date: January 19, 2016.

a) Draw a graph on 4 points with 7 paths between point A and point B.

b) Draw a graph on 4 points with 10 paths between point A and point B that must go through point C.

c) Draw a graph on any number of points that has 16 paths connecting point A and point B. What is the fewest number of edges that must be drawn to build such a graph? What is the most number of edges that can be drawn to build such a graph?

d) How many ways are there to climb 22 stairs, if you can only climb a prime number of stairs with each step?