

PARAMETRIC EQUATIONS

1. WARMUP PROBLEMS

- (1) What is the difference between the sum of the first 2018 even counting numbers and the sum of the first 2018 odd counting numbers?
- (2) How many non-congruent triangles with perimeter 6 or 7 have integer side lengths?
- (3) What is the probability that a randomly drawn positive factor of 60 is less than 7 and not prime?
- (4) A set S of points in the xy -plane is symmetric about the origin, both coordinate axes, and the line $y = x$. If $(2, 1)$ is in S , what is the smallest number of points in S ?
- (5) A square and an equilateral triangle have the same perimeter. Let A be the area of the circle circumscribed about the square and B the area of the circle circumscribed around the triangle. Find A/B .
- (6) Let n be a 5-digit number, and let q and r be the quotient and the remainder, respectively, when n is divided by 100. For how many values of n is $q + r$ divisible by 11?
- (7) The graph of the polynomial

$$P(x) = x^5 + ax^4 + bx^3 + cx^2 + dx + e$$

has five distinct x -intercepts, one of which is at $(0, 0)$. Which coefficient cannot be zero?

- (8) Objects A and B move simultaneously in the coordinate plane via a sequence of steps, each of length one. Object A starts at $(0, 0)$ and each of its steps is either right or up, both equally likely. Object B starts at $(5, 7)$ and each of its steps is either to the left or down, both equally likely. What is the probability that the objects meet?

2. PARAMETRIC PROBLEMS

- (1) What shape is traced out by the function $f(t) = (2 \cos(t), 2 \sin(t))$ for $0 \leq t \leq 2\pi$? What about $f(t) = (2 \cos(t) + 3, 2 \sin(t) + 4)$, $f(t) = (3 \cos(t), \sin(t))$, or $f(t) = (\cos(2t), \sin(2t))$?
- (2) Where do $f(t) = (\cos(t), \sin(t))$ and $g(t) = (\sin(t), \cos(t))$ intersect? How about $f(t) = (\cos(t), \sin(t))$ and $f(t) = (\cos(-t), \sin(-t))$?
- (3) Consider two particles moving along the real number line, whose positions are given by $s(t) = 3 - 2t$ and $r(t) = 4t + 1$. At what point do the particles intersect?
- (4) Write the equation for $(2 - 3t, 4 + t)$ in point-slope form.
- (5) Write a parametric equation for the line $y = 2x + 1$.
- (6) Write two different equations for the line between $(-2, 3)$ and $(6, 5)$.
- (7) Write a parametric equation for the line through the origin that makes an angle of 45° with the x axis. What if the angle is 30° instead?
- (8) What is the distance from the point $(2, 3)$ to the line described by $y = 2x - 1$? How about to $y = 2x - 1$?
- (9) What is the distance from the point $(5, -2)$ to the line described by $(4t, t - 3)$?
- (10) Find the intersection of the line $(4 - t, 2t - 1, -3t)$ and the line $(4t - 3, 1 - 2t, t + 1)$.