COUNTING MODELS

CROSSROADS ACADEMY
AMC–8 PREPARATION

1. Binomial Coefficient Practice

I) An electrician is testing 7 different wires. For each test, the electrician chooses 2 of the wires and connects them. What is the least number of tests that must be done so that every possible pair of wires is tested?

II) In how many different ways can 8 students sit at a round table?

III) A school has unlimited copies of 4 different history books, 3 different science books, and 10 different math books. Each student in the school must receive 2 different history books, 1 science book, and 4 math books and no two students are allowed to have the same collection of books. What is the maximum number of students that can be enrolled in the school?

IV) How many even four-digit numbers greater than 7,000 can be formed using the digits 3, 4, 5, 7, and 9?

V) How many 3–digit integers have no fours and at least one eight? What if there must be exactly one eight?

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VI) A room has 5 doorways. A student enters the room and then leaves by a different doorway. In how many ways can this be done? What if there are 7 doorways? n doorways?

VII) A rising number, like $14,589$, is an integer where each digit is larger than the one on its left. How many 4 digit rising numbers exist? How many d digit rising numbers exist?

VIII) A falling number, like $98,541$, is an integer where each digit is smaller than the one on its left. How many 3 digit falling numbers exist? How many d digit falling numbers exist?

IX) How many rectangles exist in a rectangular grid with 3 vertical lines and 2 horizontal lines? What if there are 3 horizontal lines? or 4? or 5? or n?

X) How many rectangles exist in a rectangular grid with m vertical lines and n horizontal lines?

2. Path Counting

XI) How many paths (following the arrows) are there from point S to point F?

XII) How many paths (following the arrows) are there from point S to point F?

XIII) How many paths (using only right and up steps) are there from point S to point F?

XIV) How many paths (using only right and up steps) are there from point S to point F that also pass through point A and point B?

3. Counting Models (Challenging!)

XV) Use the $1 \times n$ tiling model to explain why $\sum_{k=0}^{n} \binom{n}{k} = 2^n$.

XVI) Use a counting model to explain why $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$.

XVII) Use the committee selection model to explain why $k\binom{n}{k} = k\binom{n-1}{k-1}$.

XVIII) Use the domino and square tiling model to explain why $f_{m+n} = f_m f_n + f_{m-1} f_{n-1}$.

XIX) Use a counting model to explain why $\sum_{k=0}^{n} \binom{n-k}{k} = f_n$.