# PATH COUNTING

CROSSROADS ACADEMY
MATHCOUNTS PREPARATION

I) If a king begins at the lower left square of a standard chessboard, how many paths can he take to the upper right square if he can only move up or right? Can you find a formula for a general $n \times n$ chessboard?

II) If a knight begins at the lower left square of a standard chessboard, how many paths can he take to the upper right square if he can only move up and right or right and up? Can you find a formula for a general $n \times n$ chessboard?

III) If a rook begins at the lower left square of a $2 \times 4$ chessboard, how many paths can he take to the upper right square if he can only move up or right? Can you find a formula for a general $2 \times n$ chessboard?

IV) Consider paths moving only up or right on a $7 \times 3$ grid, beginning at the lower left square. What is the smallest positive integer that does not occur as a possible number of paths to a point?

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a) Draw a graph on 4 points with 7 paths between point $A$ and point $B$.

b) Draw a graph on 4 points with 10 paths between point $A$ and point $B$ that must go through point $C$.

c) Draw a graph on any number of points that has 16 paths connecting point $A$ and point $B$. What is the fewest number of edges that must be drawn to build such a graph? What is the most number of edges that can be drawn to build such a graph?

d) How many ways are there to climb 22 stairs, if you can only climb a prime number of stairs with each step?