INVOLUTIONS AND PIE

CROSSROADS ACADEMY
MATHCOUNTS PREPARATION

I) Consider the function $f(x) = \dfrac{-1}{x}$. What is $f(f(f(f(f(f(f(f(3)))))))$?

II) Consider the function $f(x) = 17 + \dfrac{1}{x - 17}$. What is $f(f(f(f(f(f(f(f(1)))))))$?

III) Consider all of the subsets of the set {apple, bananana, orange, grapefruit}. What is the number of subsets with even elements minus the number of subsets with odd elements? What happens if we add tangerine to the original set? Can you write a formula or equation for the general form of this problem?

IV) Consider all of the subsets of the set {apple, bananana, orange, grapefruit}. Count the subsets in the following way: if a subset has k elements and k is even, add k, if k is odd, add $-k$? What is the final count? What happens if we add tangerine to the original set? Can you write a formula or equation for the general form of this problem?

Date: January 19, 2016.

a) If we draw the top card of a randomly shuffled 52 card deck, what is the probability the card is a heart of a face card?

b) How many of the first $1,000,000$ positive integers are both a square and a cube? How many of the first $1,000,000$ positive integers are either a square or a cube? How many of the first $1,000,000$ positive integers are neither a square nor a cube?

c) How many positive integers no larger than 1000 are divisible by all of $4, 5$, and 6? How many positive integers no larger than 1000 are divisible by at least one of $4, 5$, or 6? How many positive integers no larger than 1000 are divisible by none of $4, 5$, and 6?

d) Early in the morning, your favorite bakery has 115 maple donuts, 116 chocolate donuts, and 117 jelly donuts. How many ways are there to purchase a dozen donuts if you must buy at least 2 of each type? Later in the afternoon, your favorite bakery is down to 5 maple donuts, 6 chocolate donuts, and 7 jelly donuts. How many ways are there to purchase a dozen donuts if you must buy at least 2 of each type?