COMPLEX ARITHMETIC

1. Complex Arithmetic

(1) For what values of k is the product $(3+2i)(4+ki)$ a real number? How about $(3+2i)(4-ki)$?

(2) Solve the equation $\dfrac{z-3i}{z+3} = 2$ for z.

(3) What are the possible values of $i^n + i^{-n}$ if n is allowed to be any integer?

(4) Find the real solution to $|z-3| = |z+2i|$.

(5) What complex number satisfies $z + |z| = 3 - 10i$?

(6) What complex number satisfies $z^2 = 21 - 20i$?

(7) What is the simpler formula for $\sum_{n=1}^{4k} ni^n$ in terms of k?

(8) If the function $f(z) = \dfrac{1}{\overline{z}}$ what is $f(f(3-7i))$?

(9) Write $(3+\sqrt{3}i)$, $2\sqrt{3}-6i$, and $(3+\sqrt{3}i)(2\sqrt{3}-6i)$ in exponential form.

(10) If x and $x-i$ are both n^{th} roots of unity, what are the possible values of n?

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2. Counting Probability Problems

(1) What is the probability that the top three cards of a shuffled deck are red?

(2) What is the probability that the top four cards of a shuffled deck are face cards?

(3) How many five card poker hands have at least three face cards?

(4) How many five card poker hands have no repeated ranks?

(5) What is the probability that there are no 3's in the top half of a shuffled deck?

(6) What is the expected value of the product of two six–sided dice?

(7) What is the expected number of rolls of a six–sided die until you roll a 4?

(8) What is the expected number of rolls of a six–sided die until you roll all the numbers at least once?

(9) What is the probability that the product of three rolls of a six–sided die is prime?