MORE PARITY PROBLEMS

CROSSROADS ACADEMY
MATHCOUNTS PREPARATION

I) Consider a knight moving on a 8×8 chessboard. What is the fewest number of moves required for moving from square a1 to a2? What is the fewest number of moves required for moving from a1 to h8? How about from e5 to b7?

II) Can you construct a path with an odd number of moves for a knight to begin at c4 and return to c4?

III) 100 checkers are placed in a row. You can exchange any pair of checkers that has exactly one checker between them, for example you could exchange the 13th and 15th checker. Is it possible to reverse the order of the checkers by performing this operation less than 2016 times?

IV) Place a knight on each square of a 7×7 chessboard. Is it possible for each knight to simultaneously make a legal move so that each knight ends up on a different square? What if it is an 8×8 board?

V) Show that for any set of three integers, we can find two of them whose average is also an integer.

Date: March 29, 2016.

a) A king is placed somewhere on a 4×4 chessboard. Two players take turns moving the king around the chessboard. In addition, the king is not allowed to return back to the square it just came from. The winner of the game is the first person who moves the king to any square it has already visited. Which player can always win this game? What if the chessboard is smaller? larger?

b) A graph is a collection of points on the plane with line segments connecting them (with at most one line segment connecting each pair of points). Show that in any graph there must be an even number of points that are connected to an odd number of other points.