**Introduction:** When analyzing redistricting from a discrete perspective we represent a plan by assigning each node of the dual graph a color to represents a given district. Properties of each district can then be evaluated by summing up the values assigned to each node of the corresponding color. The activities detailed here will help us build up some intuition for the complexity of this problem, even on small scale examples.

We'll start by looking at how to build some valid assignments on simple graphs, preserving contiguity and (at least approximately!) population balance. A district is said to be contiguous if it is possible to travel between any two nodes assigned to the district without ever leaving the district (that is, only travel along edges whose endpoints are the same color). Population balance is usually computed with respect to the "ideal population" of a district, which is simply the sum of the population of each node divided by the number of districts. This allows us to compute a deviation-from-ideal value for each district, but deciding how to aggregate these deviations to assign a score to the plan as a whole is not a settled question.

**Activity:** The second page of this document has some example 'dual graphs' that you could print out. I've also had success doing similar activities using whiteboards or the web service Limnu. The first set of examples looks at how heterogeneous populations impact the process:

- To begin try to partition the upper right graph on the next page into three contiguous and population-balanced districts. The numbers at each intersection are the population values for the corresponding node. As you are designing your partition, try to be mindful of why you are making decisions: which nodes are you starting with? what is your target population? how will you handle deviations? does the "shape" of the district factor into your decision making?
- Next, try to partition the lower right graph into eight contiguous and population-balanced districts. What lessons from the first graph are you able to apply here? What is different?

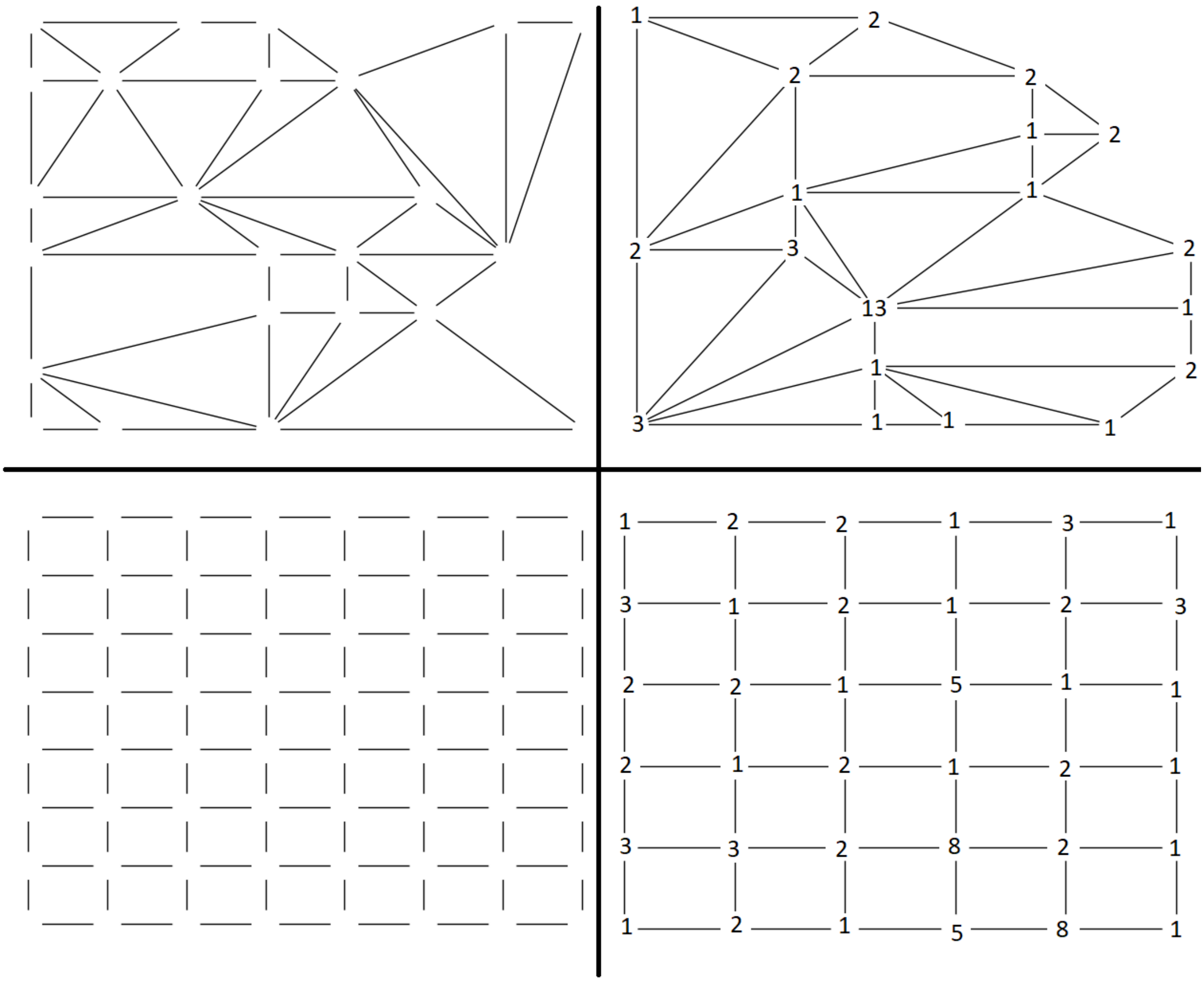
For the remaining examples, your starting point will be a $5 \times 5$ grid that you are splitting into five districts, each with five squares (like the example in the middle - we are assuming each square has the same number of residents). Here are some Sage interfaces with $5 \times 5$ grids: simple and advanced.

1. Build a combination of districts and votes so that purple gets 40% of the votes but wins no districts.
2. Construct a plan and vote distribution so that the votes are 12 - 13 but one party wins 4 districts
3. Each of you construct your own districting plan. Then, try to assign votes to the other person's plan in such a way that pink wins as many districts as possible with 40% of the votes.
4. Each of you construct your own vote distribution with at least 10 pink squares, without a plan. Then, try to creates districts over the other person's votes that get win as many districts for pink as possible. How many different electoral outcomes can you get from their vote distribution?
5. **Math Question:** How many ways are there to do this?

**Online Version:** Now that we have built up some intuition by hand, let's try out some more complete analysis using a simple computational tool. Head to `mggg.org/metagraph` and experiment with the interactives on the $4 \times 4$, $5 \times 5$, and $7 \times 7$ pages. Notice that you can input your examples from parts 3 and 4 above into the $5 \times 5$ page (the last interactive) to see if you did find the extreme examples of vote assignment.

**Discussion Questions:** Once you've completed the exercise, use the following questions to reflect on this exercise (we'll also discuss them together in a little bit):

1. Which pair of nodes did you group together first?
2. Is your plan 'fair'? How could you determine this?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